RADLEY

Scholarship Examination

# MATHEMATICS II

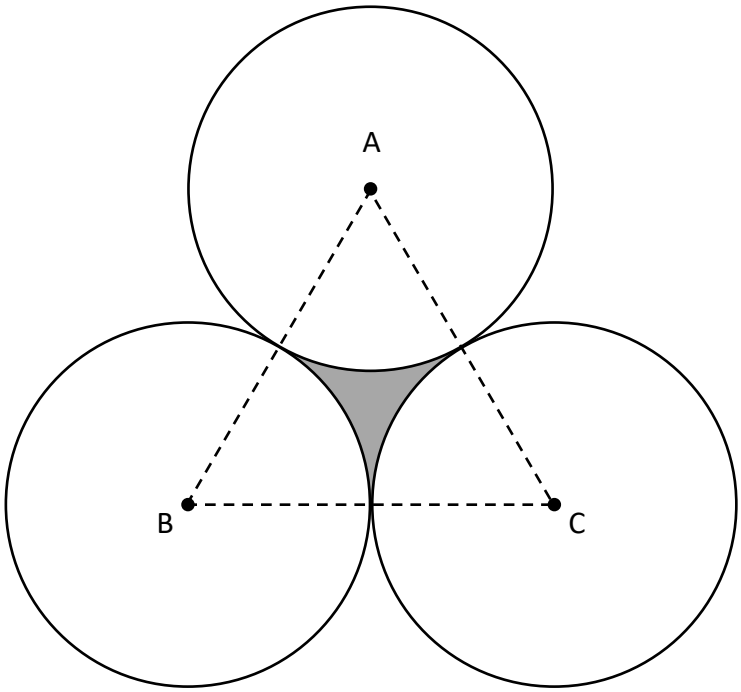
March 2020

Time allowed – 1 hour

**Show all working**

**Calculators can be used**

1. The diagram shows three circles, each of radius 1 cm, with centres A, B and C. Each circle touches the other two.




a. What sort of triangle is ABC?

b. Show that the area of triangle ABC is $\sqrt{3}$ cm$^2$.

c. Calculate the area of the shaded region.

d. Hence prove that $\pi^2 < 12$.

2. a. Solve the following simultaneous equations for $a$ and $b$:

$$\left.\begin{aligned} a + 2b &= 2 \\ 3a + b &= 10 \end{aligned}\right\}$$

b. Use your answers to part a. to solve the following equations for $x$ and $y$:

$$\left.\begin{aligned} 2^{x+2y} &= 2^{20} \\ 5^{5x+y} &= 25^{x+50} \end{aligned}\right\}$$

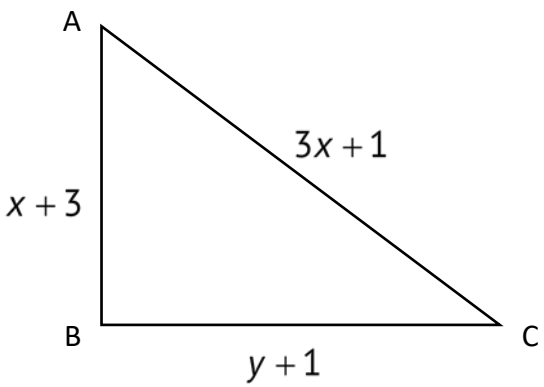
3. Two furniture stores are running *double discount* sales. SFD offers a discount of 60%, followed by another discount of 10% on that discounted price. CSC offers a discount of 50%, followed by another discount of 25% on that discounted price.

   a. Explain why a sofa with an original price of £800 will cost £288 at SFD.
   b. How much will the same sofa cost at CSC?

   CSC decides to change its discount structure. It wants to offer exactly the same overall percentage savings as SFD. It proposes to offer a discount of $x$%, followed by another discount of $x$% on that discounted price.

   c. Write down an equation for $x$, and hence solve it to find the value of $x$.

4.




The diagram shows a right-angled triangle ABC, where AB = $(x + 3)$cm, BC = $(y + 1)$cm, and CA = $(3x + 1)$cm. The right angle is at B. The perimeter of the triangle is 24cm.

a. Show that:

   (1) $y = 19 - 4x$

   (2) $8x^2 = y^2 + 2y + 9$

b. By substituting (1) into (2), and solving for $x$, find the value of $x$ and the value $y$.

5. For any positive integer, $n$, by $n!$ we mean the product of the successive positive integers between 1 and $n$ inclusive. So, for example, $3! = 1 \times 2 \times 3 = 6$.

   a. Find the value of each of the following:

      i. $6!$

      ii. $5!$

      iii. $\dfrac{6!}{5!}$

   b. Show that $\dfrac{n!}{(n-1)!} = n$

   c. Similarly, simplify:

      i. $\dfrac{(n-1)!}{(n-2)!}$

      ii. $\dfrac{n!}{(n-2)!}$

   d. Using algebra, solve the equation $\dfrac{n!}{(n-1)!} + \dfrac{(n-1)!}{(n-2)!} = \dfrac{n!}{(n-2)!} - 1$

6. a. Using each of the digits 1, 2, 3 once each, it is possible to write down six different three digit numbers. Write down these six numbers.

   b. How many of these three digit numbers are greater than 200?

   c. Using each of the digits 1, 2, 3, 4 once each, how many different four digit numbers is it possible to write down that are greater than 3000?

   d. Using each of the digits 1, 2, 3, 4, 5 once only, how many different five digit numbers is it possible to write down that are greater than 40000?